

Exercise 53

Solve each equation for x .

(a) $2^{x-5} = 3$

(b) $\ln x + \ln(x-1) = 1$

Solution**Part (a)**

$$2^{x-5} = 3$$

Take the natural logarithm of both sides.

$$\ln 2^{x-5} = \ln 3$$

Bring the exponent down in front.

$$(x-5) \ln 2 = \ln 3$$

$$x-5 = \frac{\ln 3}{\ln 2}$$

Therefore,

$$x = 5 + \frac{\ln 3}{\ln 2} \approx \pm 6.585.$$

Part (b)

$$\ln x + \ln(x-1) = 1$$

The logarithms have the same base, so they can be combined.

$$\ln[x(x-1)] = 1$$

Exponentiate both sides.

$$e^{\ln[x(x-1)]} = e^1$$

$$x(x-1) = e$$

$$x^2 - x - e = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-e)}}{2(1)} = \frac{1 \pm \sqrt{1 + 4e}}{2}$$

Only the positive sign works because $x > 0$ is a requirement, looking at the starting equation. Therefore,

$x = \frac{1 + \sqrt{1 + 4e}}{2} \approx 2.223$

~~$$x = \frac{1 - \sqrt{1 + 4e}}{2} \approx -1.223.$$~~